Sevilla Carpets of Deterministic Non-cooperative P Systems

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Summary. A Sevilla carpet is a graphical representation of the sequence of multisets of rules applied at every step of the evolution of a P system. This paper is devoted to the research of the shape of Sevilla carpets. The case of deterministic non-cooperative systems is considered, and a characterization of Sevilla carpets is given.

1 Introduction

P systems were introduced by Gh. Păun in [2] as distributed parallel computing devices of biochemical inspiration. These systems are inspired from the structure and the functioning of a living cell. The cell is considered as a set of compartments (membranes) nested one in another and containing objects and evolution rules. The base model specifies neither the nature of these objects, nor the nature of rules. Numerous variants specify these two parameters by obtaining a lot of different models of computing; we refer to [5] for a comprehensive bibliography.

Various approaches are used to study these systems. Like in the classical theory of formal languages, first, the computational power with respect to different parameters is studied. There are many such decidability and undecidability results for most of the classes of P systems. After that, complexity classes are introduced and investigated.

Recently, efforts to build models of different phenomena based on P systems were made. In this case, the evolution dynamics of the system is studied and no more the generated language. *Sevilla carpets* permit to have a first overview of the dynamic evolution of the system [1]. More exactly, a Sevilla carpet is a time/frequency diagram which shows how many times a rule is applied at some moment of time. Potentially, Sevilla carpets may be used to locate critical sections of the execution and to facilitate an optimization of the system.

In this article, we consider a particular class of P systems, *deterministic non-cooperative P systems*, and we study Sevilla carpets associated to this model. In this case, we show that we may reduce the problem to growth functions of D0L systems. We give several examples that illustrate this reduction and we present at the end some remarks and open questions.

2 Preliminaries

We will use the following notations. For a string $w \in O^*$ and a symbol $a \in O$ we denote the number of occurrences of a in w by $|w|_a$ and the length of w by $|w|$; for a string w and a number i, $1 \leq i \leq |w|$, we denote the i-th symbol of w by $w(i)$. We write the components of a vector $v \in \mathbb{N}^k$ as $v[i]$, $1 \leq i \leq k$.

The basic model of a P system is the *transitional* one. A (non-cooperative) transitional P system (with m membranes) is a tuple

$$\Pi = (O, \mu, w_1, \cdots, w_n, R_1, \cdots, R_n, i_0),$$

where O is a finite set of objects, μ is the membrane structure, the string w_i specify multisets of objects initially present in region i, and R_i is a finite set of rules associated to region i. The rules are of the following form: $a \to u$, where $a \in O$, $u \in O \times Tar$, $Tar = \{in_j \mid 1 \leq j \leq m\} \cup \{here, out\}$. i_0 is the output region (or 0 if it is the environment) and it is irrelevant for studying the dynamics of the P system. The execution of a rule as above replaces one occurrence of a with the multiset of objects specified by u. The set Tar represents the target indications: the resulting objects immediately move to the corresponding location (*here* corresponds to the same region, *out* corresponds to the immediately external region, and in_j corresponds to region j, where j is immediately internal).

The rules are executed in the maximally parallel way, non-deterministically. For the non-cooperative case this means all objects that have rules assigned to them are simultaneously processed by these rules. A configuration is halting if no rules are applicable. The result of a halting computation (sequence of configurations) is the number of objects (or the multiplicities of all objects, or the sequence of objects if $i_0 = 0$) present in the output region.

Since the power of this model is limited (only Parikh sets of context-free languages can be generated), there are various extensions introduced and studied: by the rules used – cooperative, catalytic, communicational, etc., by changing membrane structure – rules for dissolving membranes, dividing membranes, creating membranes, by changing membranes – controlling membrane permeability, membrane polarizations, others – sequential use of (some) rules, assigning energy to rules or membranes, etc. We refer to [3] for a systematic survey, and to [5] for a comprehensive bibliography.

In this article we will consider none of these extensions. In fact, we will consider one restriction, namely the determinism. The system is called *deterministic* if there is only one computation possible. Notice that for non-cooperative systems determinism implies that no two rules of the same region j act on the same object a (unless a can never appear in this region j, and then such rules can be removed without restricting the generality much).

Let us assume that all rules in a P system Π have distinct labels, and let R be the set of these labels. A *Sevilla carpet* of Π can be defined as a mapping $n : \mathbb{N} \times R \to \mathbb{N}$, where $n(s, r)$ is the number of times the rule r is applied at step s. For an easier representation of the results, we assumed that the first step of the computation is step 0 (otherwise s should have been replaced by $s - 1$).

3 Reducing the Problem to L Systems

Consider a deterministic non-cooperative P system. We have assumed that all rules act either on different objects, on in different regions; hence, every object-region pair is associated to at most one rule. Notice that the objects produced in the regions where they do not evolve do not influence the carpet, so assuming no idle objects does not restrict the generality.

Take such a P system $\Pi = (O, \mu, w_1, \cdots, w_m, R_1, \cdots, R_m)$, assume that the rules $R_1, \cdots, R_m$ are uniquely labelled, and the set of labels is V. Let us define a mapping $\rho : \{1, \cdots, m\} \times O \to V \cup \{\lambda\}$ as

$$\rho(i, a) = \begin{cases} \lambda & \text{if } \; R_i \text{ has no rule } a \to \lambda, \\ l & \text{if } \; l \text{ is a label of some rule } (a \to \lambda) \in R_i \end{cases}$$

(recall that there exists at most one rule of the form $a \to \lambda$).

Construct a deterministic L system $G = (V, w_r, P)$, where

$$w_r = \prod_{i=1}^{m} \prod_{j=1}^{|w_i|} \rho(i, w_i(j)),$$

$$P = \{l \to h(l) \mid l = \rho(i, a) \neq \lambda, \; h(l) = \prod_{j=1}^{|\alpha|} \rho(i, \alpha(j)),$$
$$(a \to \alpha) \in R_i, \; a \in O, \; 1 \leq i \leq m\}.$$

Consider the evolutions of Π and G; notice that each (copy of an) object in G corresponds one to (application of a) rule in Π. In this way, in the deterministic non-cooperative case the characterization of Sevilla carpets can be reduced to the study of the (Parikh vector) growth functions of $D0L$ systems. The converse is quite simple: given a $D0L$ system, a P system with the same behavior (carpet) can be obtained just by putting the $D0L$ in one membrane.

For the rest of the paper, let us introduce another useful representation. Suppose that the elements of V are ordered ($|V| = k$, $V = \{l_1, \cdots, l_k\}$). Let $v = (|w_r|_{l_1}, \cdots, |w_r|_{l_k})$ be the vector of initial multiplicities. Let $M = (n_{i,j})_{i,j\in\{1,\cdots,k\}}$ be the transition matrix. The elements of M are defined as $n_{i,j} = |h(l_i)|_{l_j}$.

Notice that $Ps(w_r) = v$, $Ps(h(w_r)) = vM$, etc., so $Ps(h^s(w_r)) = vM^s$, $s \geq 0$. Recall that the elements of Sevilla carpet of the original system (number of applications of rule l_i at time $s \geq 0$) correspond to the components of vM^* (i.e., to the numbers $(vM^s)[i]$, $1 \leq i \leq k$, $s \geq 0$).

4 Vector Growth Functions

This approach can be found in some university algebra textbooks, or in [4]. For a transition matrix M, the characteristic equation $det(M-pE)$ needs to be solved to obtain the eigenvalues $p_1, \cdots, p_e$ with multiplicities $m_1, \cdots, m_e$, $m_1+\cdots+m_e = k$. The general solution of the growth function is of the form $vM^s = (v_{1,1} + \cdots + v_{1,m_1}s^{m_1-1})p_1^s+\cdots+(v_{e,1}+\cdots+v_{e,m_e}s^{m_e-1})p_e^s$. The coefficients $v_{1,1}, \cdots, v_{e,m_e}$ can be determined from the equations obtained from the first k steps ($0 \leq s \leq k-1$).

5 The Case of Two Rules

Consider a P system with two objects: $\Pi = (\{a,b\}, [_1\ ]_1, a^{v[1]}b^{v[2]}, R)$, $R = \{p_1 : a \to a^{n_{1,1}}b^{n_{1,2}},\ p_2 : b \to a^{n_{2,1}}b^{n_{2,2}}\}$. It corresponds to a $D0L$ system where P consists of rules of R, where a is replaced by p_1 and b is replaced by p_2. (Notice that $\Pi' = (\{a\}, [_1\ [_2\ ]_2\ ]_1, a^{v[1]}, a^{v[2]}, R)$ with $R_1 = \{p_1 : a \to a^{n_{1,1}}a_{in}^{n_{1,2}}\}$, $R_2 = \{p_2 : a \to a_{out}^{n_{2,1}}a^{n_{2,2}}\}$ would correspond to a $D0L$ system where P consists of rules of R, where a in region 1 is replaced by p_1 and a in region 2 is replaced by p_2. The $D0L$ system corresponding to Π and Π' is the same: $G = (\{p_1, p_2\}, w, P)$, $P = \{p_1 \to p_1^{n_{1,1}}p_2^{n_{1,2}},\ p_2 \to p_1^{n_{2,1}}p_2^{n_{2,2}}\}$.) The transition matrix is $(n_{i,j})_{i,j\in\{1,2\}}$.

The characteristic equation is $p^2-(n_{1,1}+n_{2,2})p+(n_{1,1}n_{2,2}-n_{1,2}n_{2,1}) = 0$. There are two cases, depending on whether $D = (n_{1,1} + n_{2,2})^2 - 4(n_{1,1}n_{2,2} - n_{1,2}n_{2,1})$ is zero or not. Notice that $D = (n_{1,1} + n_{2,2})^2 - 4(n_{1,1}n_{2,2} - n_{1,2}n_{2,1}) = (n_{1,1} - n_{2,2})^2 + 4(n_{1,2}n_{2,1})$ cannot be negative with nonnegative values of coefficients $n_{i,j}$, so we will always obtain the real roots of the characteristic equation.

Case 1: $p_1 = p_2 = p = (n_{1,1} + n_{2,2})/2$, $vM^s = (v_{1,1} + v_{1,2}s)p^s$. Finding the coefficients: $v_{1,1} = v$, $p(v_{1,1} + v_{1,2}) = vM$, thus $v_{1,1} = v$, $v_{1,2} = v(M - pE)/p$ ($p_1 = p_2 = 0$ is a special case, but in this case everything is erased in one step).

Putting it all together, if $n_{1,1} = n_{2,2}$ and $n_{1,2}n_{2,1} = 0$, then

$$vM^s = v\left(1 + \frac{2M - (n_{1,1} + n_{2,2})E}{n_{1,1} + n_{2,2}}\ s\right)\left(\frac{n_{1,1} + n_{2,2}}{2}\right)^s$$

Componentwise:

$$(vM^s)[1] = \Big(v[1] + \frac{2v[1]n_{1,1} + 2v[2]n_{1,2} - (n_{1,1} + n_{2,2})}{n_{1,1} + n_{2,2}}\ s\Big)\Big(\frac{n_{1,1} + n_{2,2}}{2}\Big)^s$$
$$(vM^s)[2] = \Big(v[2] + \frac{2v[1]n_{2,1} + 2v[2]n_{2,2} - (n_{1,1} + n_{2,2})}{n_{1,1} + n_{2,2}}\ s\Big)\Big(\frac{n_{1,1} + n_{2,2}}{2}\Big)^s$$

Example 1. $\Pi = (\{a,b\}, [_1\ \ ]_1, abbbbb, R)$, $R = \{p_1 : a \to aa,\ p_2 : b \to bb\}$. $p_1 = p_2 = p = 2$. $vM^s = (v_{1,1} + v_{1,2}s)2^s$. Coefficients: $v_{1,1} = (1,5)$, $v_{1,2} = (1,5) * (2E - 2E) = (0,0)$. Answer:

$$vM^s = (2^s, 5 \cdot 2^s).$$

Example 2. (Polynomial growth) $\Pi = (\{a,b\}, [_1\ \ ]_1, aabbb, R)$, $R = \{p_1 : a \to abbbb,\ p_2 : b \to b\}$. $p_1 = p_2 = p = 1$. $vM^s = (v_{1,1} + v_{1,2}s)1^s$. Coefficients: $v_{1,1} = (2,3)$, $v_{1,1} + v_{1,2} = (2,11)$, so $v_{1,1} = (0,4)$ Answer:

$$vM^s = (2, 3 + 4s).$$

Case 2: $p_1, p_2 = \frac{n_{1,1}+n_{2,2}\pm\sqrt{(n_{1,1}+n_{2,2})^2-4(n_{1,1}n_{2,2}-n_{1,2}n_{2,1})}}{2}$, $vM^s = v_{1,1}p_1^s + v_{2,1}p_2^s$. Coefficients: $v_{1,1} + v_{2,1} = v$, $p_1v_{1,1} + p_2v_{2,1} = vM$. (Notice that coefficients $1, 1, p_1, p_2$ are the same for solving for $v_{i,1}[1]$ and for solving for $v_{i,1}[2]$) Solution: $\delta = p_2 - p_1$, $D_1 = -v(M - p_2E)$, $D_2 = v(M - p_1E)$, so $v_{1,1} = D_1/\delta$, $v_{2,1} = D_2/\delta$.

Putting it all together, if $n_{1,1} \neq n_{2,2}$ or $n_{1,2}n_{2,1} \neq 0$, then

$$vM^s = \frac{v}{4\sqrt{n_{1,1} - n_{2,2})^2 - 4(n_{1,2}n_{2,1})}} \times \Bigg(\Big(2M - \big(n_{1,1} + n_{2,2} - \sqrt{(n_{1,1} - n_{2,2})^2 - 4(n_{1,2}n_{2,1})}\big)E\Big) \Big(\frac{n_{1,1} + n_{2,2} + \sqrt{(n_{1,1} - n_{2,2})^2 - 4(n_{1,2}n_{2,1})}}{2}\Big)^s - \Big(2M - \big(n_{1,1} + n_{2,2} + \sqrt{(n_{1,1} - n_{2,2})^2 - 4(n_{1,2}n_{2,1})}\big)E\Big) \Big(\frac{n_{1,1} + n_{2,2} - \sqrt{(n_{1,1} - n_{2,2})^2 - 4(n_{1,2}n_{2,1})}}{2}\Big)^s\Bigg)$$

Componentwise:

$$(vM^s)[i] = \frac{1}{4\sqrt{n_{1,1} - n_{2,2})^2 - 4(n_{1,2}n_{2,1})}} \times \Bigg(\Big(2v[1]n_{i,1} + 2v[2]n_{i,2} - \big(n_{1,1} + n_{2,2} - \sqrt{(n_{1,1} - n_{2,2})^2 - 4(n_{1,2}n_{2,1})}\big)\Big) \Big(\frac{n_{1,1} + n_{2,2} + \sqrt{(n_{1,1} - n_{2,2})^2 - 4(n_{1,2}n_{2,1})}}{2}\Big)^s$$

$$- \Big(2v[1]n_{i,1} + 2v[2]n_{i,2} - \big(n_{1,1} + n_{2,2} + \sqrt{(n_{1,1} - n_{2,2})^2 - 4(n_{1,2}n_{2,1})}\big)\Big)$$
$$\Big(\frac{n_{1,1} + n_{2,2} - \sqrt{(n_{1,1} - n_{2,2})^2 - 4(n_{1,2}n_{2,1})}}{2}\Big)^s\Big), \ 1 \le i \le 2.$$

Example 3. $\Pi = (\{a, b\}, [_1 \ ]_1, a, R)$, $R = \{p_1 : a \to bb,\ p_2 : b \to aaa\}$.

$D = -4(-2\cdot3)$. $p_1, p_2 = \pm2\sqrt{6}/2$. $vM^s = v_{1,1}\sqrt{(6)}^s + v_{2,1}(-\sqrt{6})^s$. Coefficients: $v_{1,1} + v_{2,1} = (1, 0)$, $(v_{1,1} - v_{2,1})\sqrt{6} = (0, 2)$, hence, $v_{1,1} = (1/2, \sqrt{6}/3)$, $v_{1,1} = (1/2, -\sqrt{6}/3)$, Answer:

$$vM^s = (\sqrt{6})^s((1/2, \sqrt{6}/3) + (-1)^n(1/2, -\sqrt{6}/3)).$$

Thus, at step s rule p_1 will be applied $6^{s/2}(1 + (-1)^n)/2$ times, and rule p_2 will be applied $6^{(s-1)/2}(1 - (-1)^n)$ times.

Example 4. $\Pi = (\{a, b\}, [_1 \ ]_1, a, R)$, $R = \{p_1 : a \to abb,\ p_2 : b \to aabbb\}$.

$D = (1-3)^2 + 4(2\cdot2) = 20$. $p_1, p_2 = 2 \pm \sqrt{5}$. $vM^s = v_{1,1}(2 + \sqrt{(5)})^s + v_{2,1}(2 - \sqrt{5})^s$. Coefficients: $v_{1,1} + v_{2,1} = (1, 0)$, $(2+\sqrt{5})v_{1,1} + (2-\sqrt{5})v_{2,1}) = (0, 2)$, hence, $v_{1,1} = (\frac{1}{2} - \frac{1}{2\sqrt{5}}, \frac{1}{2} + \frac{1}{2\sqrt{5}})$, $v_{1,1} = (\frac{1}{\sqrt{5}}, -\frac{1}{\sqrt{5}})$, Answer:

$$vM^s = (2+\sqrt{5})^s\left(\frac{1}{2} - \frac{1}{2\sqrt{5}}, \frac{1}{2} + \frac{1}{2\sqrt{5}}\right) + (2-\sqrt{5})^s\left(\frac{1}{\sqrt{5}}, -\frac{1}{\sqrt{5}}\right).$$

These numbers actually correspond to a subsequence of the famous Fibonacci numbers ($F(0) = 0$, $F(1) = 1$, $F(i+2) = F(i+1) + F(i)$, $i \ge 0$): $vM^s = (F(3s), F(3s+1))$. The transition matrix is actually a third power of the Fibonacci one (corresponding to rules $a \to b$, $b \to ab$).

6 Conclusions

The Sevilla carpet of a deterministic non-cooperative P system is a numerical function (application multiplicity) with one argument (what rule) ranging over a finite set and the other argument (at what step) being numerical, from a potentially infinite set. With respect to the time, this function can have polynomial or exponential growth.

Considering non-deterministic systems is more difficult, since then one system defines a family of carpets (each corresponding to a computation). The behavior of cooperative systems is much more difficult to describe or to predict. Nevertheless, studying some properties of them is an interesting research topics.

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